

Mathematical Exploration with HP Prime



Solving Equations

What does it mean to solve an equation? We are looking for an equivalent expression to our original equation that is in the form: variable = value. For example, if we need to solve $3x - 17 = 22$ then we aim to find an equivalent statement by legitimate algebraic moves, that yields $x = 13$.

With a computer algebra system (CAS) students can explore these moves and generate their own methods towards solution.

There is a central premise that some people can find a little odd. Generally, we talk of the legitimate moves as “doing the same to both sides”. This builds on the premise of balance, which is just a metaphor. However, within a CAS, we can act on the whole statement at once which makes clearer how we can preserve identities. We invite you to retain an open mind!

Press CAS

There may be many past entries filling the screen. These are not a problem. If you would prefer to start with a blank screen, press Shift Clear (Esc)

Now enter the equation $3x - 17 = 22$ (Use ALPHA X for the variable. On the CAS screen it defaults to a lower case x as all CAS variables must be in lower case. For ‘=’ use Shift = (the decimal point key).

Press Enter.

Now we want to change the expression but preserve the identity. So let’s add 17 to the expression.

Press () for brackets. The cursor stays between the brackets.

Press the up arrow so that the expression we entered is highlighted.

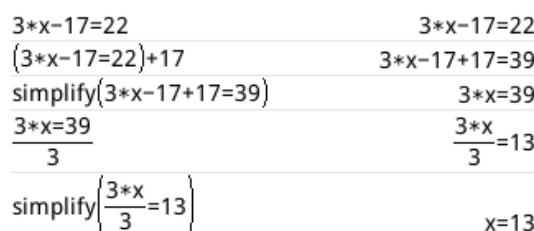
Press Copy.

Now press the right arrow to move the cursor outside the brackets.

Now type +17 and press Enter.

You can see what has been done, but you will want to simplify, so press simplif on the screen.

Now divide the expression by 3. Again enter brackets (), then use up arrow to copy the new expression, then right to get outside the brackets, then divide by 3 and press Enter. See what has happened, then simplify.



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We can check in different ways, which is good practice, to make us confident in what has happened.

So try $3 \cdot 13 - 17$ and Enter.

Then use the more efficient $\text{solve}(3x-17=22)$ and Enter.

You can type the word solve or you can use the suitcase menu, CAS then solve.

As teacher, I would work through this first example as a whole class. But importantly, I must not prompt for the steps. Whatever students suggest, we can try. If it makes things simpler, or closer to the all-important step of variable = value, then it was a useful step. In any event, we can carry on from whatever suggestions are made.

Now try something like $5x + 19 = 2x - 7$

There are many different ways to proceed, so student responses will show the many equivalent routes.

Some may be more or less efficient than others. This makes for a good discussion.

Here we show one possible route.

We have finished by checking with the solve command.

Suppose we have more than one equation.

For example $3x + 5y = 19$ and $2x - 3y = 12$.

Can we modify these to make statements of the form variable = value?

Here is one way to do this, but of course there are many ways. Students will find many blind alleys in trying to find a way that works. That is very good. They need to develop their skills and find what does not work as much as what does work. We show a clean solution, but this will not be representative.

Before we make a new statement we enclose the old one in brackets and then make some change.

CAS Statistics 2Var 16:22

$3x-17=22$	$3x-17=22$
$(3x-17=22)+17$	$3x-17+17=39$
$\text{simplify}(3x-17+17=39)$	$3x=39$
$\frac{3x=39}{3}$	$\frac{3x}{3}=13$
$\text{simplify}\left(\frac{3x}{3}=13\right)$	$x=13$
$3 \cdot 13 - 17$	22
$\text{solve}(3x-17=22)$	$\{13\}$

Sto simplif

CAS Statistics 2Var 16:28

$5x+19=2x-7$	$5x+19=2x-7$
$(5x+19=2x-7)-2x$	$5x+19-2x=2x-7-2x$
$\text{simplify}(5x+19-2x=2x-7-2x)$	$3x+19=-7$
$(3x+19=-7)-19$	$3x+19-19=-26$
$\text{simplify}(3x+19-19=-26)$	$3x=-26$

Sto simplif

CAS Statistics 2Var 16:29

$3x+19=-7$	$3x+19=-7$
$(3x+19=-7)-19$	$3x+19-19=-26$
$\text{simplify}(3x+19-19=-26)$	$3x=-26$
$\frac{3x=-26}{3}$	$\frac{3x}{3}=\frac{-26}{3}$
$\text{simplify}\left(\frac{3x}{3}=\frac{-26}{3}\right)$	$x=\frac{-26}{3}$
$\text{solve}(5x+19=2x-7)$	$\left\{\frac{-26}{3}\right\}$

Sto simplif

CAS Statistics 2Var 16:37

$3x+5y=19$	$3x+5y=19$
$2x-3y=12$	$2x-3y=12$
$(3x+5y=19) \cdot 3$	$(3x+5y) \cdot 3=57$
$\text{simplify}((3x+5y) \cdot 3=57)$	$9x+15y=57$
$(2x-3y=12) \cdot 5$	$(2x-3y) \cdot 5=60$
$\text{simplify}((2x-3y) \cdot 5=60)$	$10x-15y=60$
$(9x+15y=57) + ((2x-3y) \cdot 5=60)$	$9x+15y+(2x-3y) \cdot 5=117$

Sto simplif

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When we have two expressions that can be added to remove one variable, we can put each in brackets by finding it with the cursor and placing it with copy.

```
CAS Statistics 2Var 16:38
simplify((2*x-3*y)*5=60) 10*x-15*y=60
(9*x+15*y=57)+((2*x-3*y)*5=60)
9*x+15*y+(2*x-3*y)*5=117
simplify(9*x+15*y+(2*x-3*y)*5=117)
19*x=117
19*x=117 19*x=117
19 19
simplify(19*x=117) x=117/19
```

We can now reduce to find the value of one variable.

We now find and copy one of the original equations and edit it, replacing x with the value we have now found.

```
Sto simplif
CAS Statistics 2Var 16:44
simplify(19*x=117) x=117/19
2*(117/19)-3*y=12 234/19-3*y=12
(234/19-3*y=12)-12+3*y
234/19-3*y-12+3*y=3*y
simplify(234/19-3*y-12+3*y=3*y) 6/19=3*y
```

This is then simplified to find y .

```
Sto simplif
CAS Statistics 2Var 16:48
simplify(6/19=3*y) 2/19=3*y
6/19=3*y 2/19=3*y
3 19
simplify(2/19=3*y) 2/19=y
solve(3*x+5*y=19 AND 2*x-3*y=12,{x,y})
[[117 2]]
[19 19]
```

The solve command still works for checking purposes, but note carefully how two equations are entered and that we need to say which variables we want to solve for, as a set, in curly brackets:

$$\text{Solve}(3x + 5y = 19 \text{ and } 2x - 3y = 12, \{x, y\})$$

```
Sto simplif
```

It is interesting to let students try more complicated equations to see when new algebraic techniques are required. For example, they may be OK with $2x^2 + 3 = 29$ but may not be able to complete $x^2 + 17x - 23 = 0$. So, a new technique is needed. Could they do $3 \sin(2x + 3) + 11 = 9$ this way?

As students gain experience with the algebraic possibilities, they will get better at understanding the methods they are using. It is very important that students write the working from the CAS onto paper, using standard syntax for (hand) written algebra. Also, it is very important that they have opportunities to explain their methods and how they work to others. In the end their methods must work without the aid of the CAS, so they can validate their work by solving problems with paper and pencil alone.