Mathematical Exploration with HP Prime

Exploring Projectiles

In this activity we will explore the paths more than one projectile and an inclined plane.

We know for a projectile the horizontal equation of motion is
$x = utcos(A)$

The vertical equation is $y = utsin(A) - 0.5gt^2$

Where u is the initial velocity, A the angle of projection, g acceleration due to gravity and t the time taken.

So, we have two parametric equations with parameter t

First, we enter the equations: (note that T is the parameter so use the X,T,θ,N key or the on-screen T).

Press Apps, find Parametric, press Reset then OK, then Start

For X1(T) type ALPHA U × T × COS(ALPHA A) then OK

For Y1(T) type ALPHA U × T × SIN(ALPHA A) − 0.5 × ALPHA G × T² (use the x^2 key) then OK

Set initial values for the variables:

Press HOME, type 9.8 press STO ALPHA G ENTER

Now choose some sensible values for the other variables, (say):

30 STO ALPHA A ENTER

5 STO ALPHA U ENTER

Press Shift Home Settings and set the Angle Measure to degrees.

Now press Plot to see the projectile.

Now we can add a second projectile to explore when and where projectiles may meet. We can allow for the possibility that either starts at a different height by adding another variable to the vertical equation.

Press Symb.

Make sure the cursor is in the line for Y1(T) and press Edit.

Use the cursor to move to the beginning of the equation and add H+ to make the equation Y1(T)=H+U*T*SIN(A)-.5*G*T² and press OK

Now enter X2(T)=V*T*COS(B) and Y2(T)=J+V*T*SIN(B)-.5*G*T²

(You can use Copy, Paste and Edit, or you can just type directly).
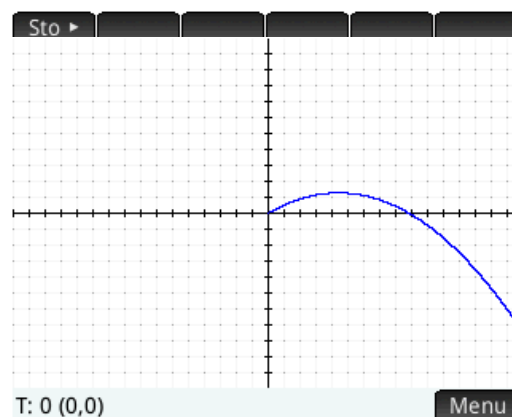
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On the home screen set initial values as before.
I would recommend starting with the same values for the second projectile as you have for the first, and setting J=1 which is the initial height of the second projectile.
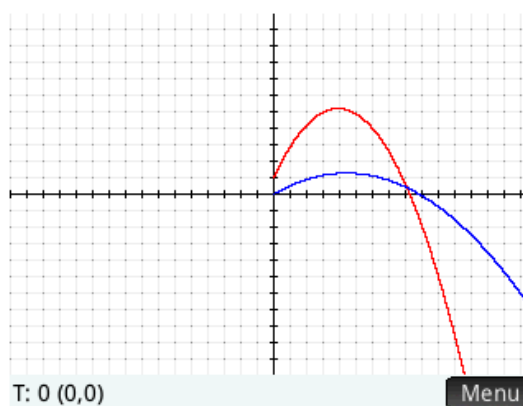
Parametric	09:58

9.8▸G	9.8
30▸A	30
10▸U	10
30▸B	30
10▸V	10
1▸J	1

Sto ▸					

Press Plot to see the paths of the two projectiles.

T: 0 (0,0)   Menu
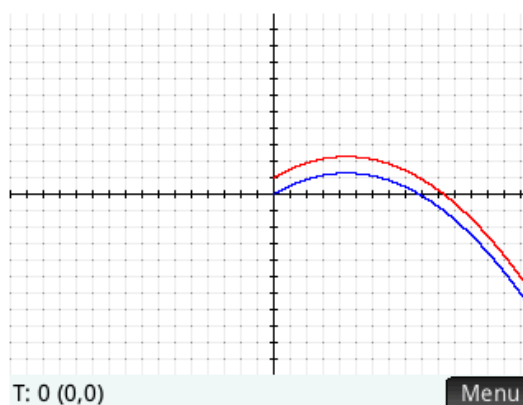
Now explore:
What initial angle for the second projectile is sufficient for it to meet the first projectile before they hit the ground?

(Keep returning to the home screen and trying new values for B using e.g. 40 STO ALPHA B ENTER)

T: 0 (0,0)   Menu

Click on the point of intersection to position the cursor, then use the + key to zoom in. Now you can see the position and time of intersection to any level of accuracy you like.

Press Menu if you cannot see the values because the menu items are in the way.

Also, use Shit + Settings and change the Number Format to (e.g.) Fixed 3 to see the values clearly.

Note: If you want to return to the initial screen view: press Menu, then Zoom, scroll down to find Decimal and press it.

T: .930 (8.057,.410)   Menu
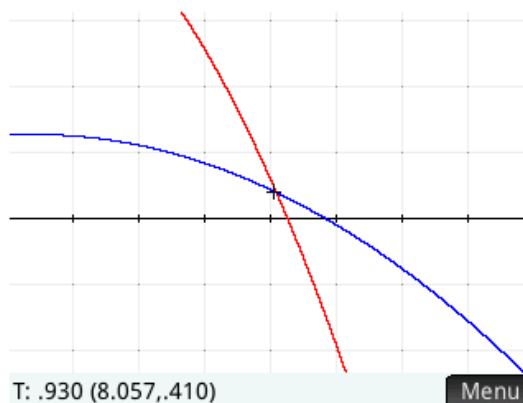
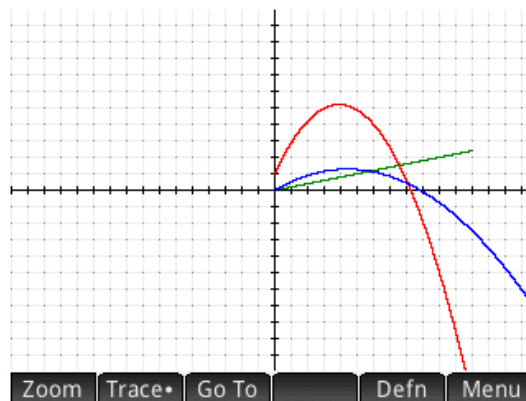Experiment to find the value of B which makes the collision exactly on the ground.

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You can also add an inclined plane to your model.

Press Symb and enter X3(T)=T and Y3(T)=K*T

In the home screen set a gradient for the plane e.g. K=0.2 using

0.2 STO ALPHA K ENTER

You can now explore the interrelationships between the variables:

Remember:

U and V are the initial speed and A and B the angle of projection of the two projectiles.

H and J are the initial height above the ground for the two projectiles.

G is acceleration due to gravity for both projectiles.

K is the gradient of the inclined plane.

One final parameter you can explore in your model is time. This is the parameter T which links the two parametric equations, so it is an independent variable.

You will have noticed that the inclined plane does not extend to the end of the screen. This is because the domain for T is pre-set, so the calculator does not have to calculate too many values of X and Y. Pressing Shift and Plot gives access to this setting.

The range of T (T Rng) is initial set form 0 to 12. We can see what happens if we go back in time by changing the initial setting to -12. (Use the +/- key not the subtract key)

Be careful not to set the range too large or the graph will take some time to draw.

Now press Plot to see the outcome.

You now have a model with two projectiles with variable initial height and a variable inclined plane able to move in positive and negative time.
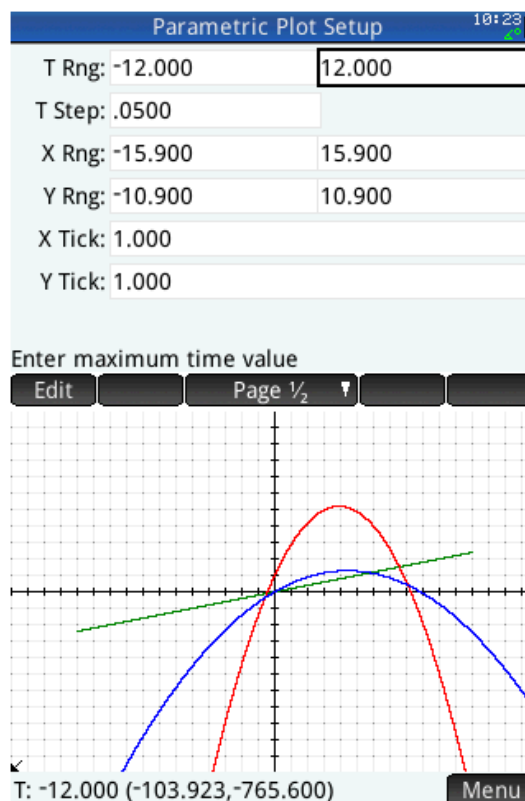
Create your own problems varying these parameters looking at the time and position of collisions between the projectiles themselves and the projectiles and the ground with varying conditions.