

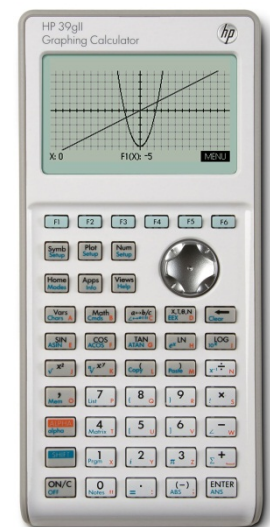
The Maths Activities Pack

Using HP Prime and HP39gII Graphing Calculators



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Introduction

Graphing calculators have been around since the late 1980s. They are easily carried, turn on instantly, are extremely reliable, yet have software as powerful as anything available on a full computer for doing mathematics. In the classroom, ICT use is often difficult to arrange, booking the room, finding enough working machines and keeping the students off the internet are ever present problems. With a class set of graphing calculators in a small, easily carried box and a software emulator on the classroom computer, the teacher can have proper technology rich lessons with minimal set up time and maximal reliability. It just works!

Hewlett Packard have been in the forefront of calculator technology since its earliest days. They are renowned for the very high physical quality of the machines together with the most thoughtfully developed software. Sometimes, teachers are worried about the complexity of graphing calculators, but HP's designs work the way maths teachers think. Ease of teacher use is built in to the design from the start.

The activities in this booklet are designed for use with the HP Prime and HP39gII with additional activities also incorporating the StreamSmart data logging device. Both of the calculators have extremely comprehensive scientific functions, including matrices and complex numbers. However, their power is seen in the range of function types for which multiple representations can be set up. There is considerable research evidence showing the improvements in students' understanding when they have access to symbolic, graphical and table representations. The HP Prime and HP39gII make this a central feature and can quickly show all three, for functions plots, polar graphs, parametric graphs and many more. The same structure even applies to one variable and two variable statistics. Sets of side by side box-and-whisker plots can be drawn as soon as the data is entered, together with the summary statistics. Students don't need to laboriously draw the charts themselves, instead they can get straight to the statistical analysis.

HP Prime has a computer algebra system (CAS). This means that it can operate on algebraic statements, working symbolically. It will factorise symbolic expressions, it will integrate and differentiate symbolically and very much more. The CAS can be disabled securely so that the machine can still be used in public examinations.

The HP39gII has a high resolution screen able to show grey scales for beautiful, smooth graphs, coupled with a fast processor and more than enough memory. Also, a sophisticated battery management system means that standard AAA batteries can last for months, meaning you can always be sure of a complete set of working machines. There are very many ease-of-use improvements which the user will see in

operation. Options will come up when they are needed. There are six SoftKeys F1 to F6, which make navigation much clearer. Also, there is a comprehensive notes system for developing classroom activities and a detailed, context sensitive help system.

HP Prime is a major advance. Taking all of the features of the HP39gII, plus the CAS system, there is now a multi-touch full colour screen for a fast and intuitive interface. Then there is extra software, a dynamic geometry system, a spreadsheet and the amazing advanced graphing app, which can graph **any** statement in x and y .

The StreamSmart device together with data logging probes plugs directly into the calculators and allows very fast, real time logging of a variety of data types. For example, students can log their displacement from the device, creating real-time distance-time graphs. There is a thermometer, a microphone, a pressure probe and a light intensity probe available, allowing for a range of experiments for STEM lessons.

The activities in this booklet are not set out as lesson plans. We assume that teachers will want to adapt them for different parts of lessons and different timings. Instead we have given details of the mathematics and instructions for conducting the activity, together with commentary on classroom use. All of these have been used by the author in perfectly ordinary secondary school classrooms.

The emphasis is on exploratory and investigative working. The power of the calculator lies in its ability to give instant feedback to ideas and to generate data very quickly. Students can generate data, make and test hypotheses about how mathematical operations and ideas work. They then test and develop their hypotheses to reach a conclusion. At this stage, the technology must be set aside and the student can practice their ideas away from the technology.

We would recommend a standard class set of one between two students, to maximise collaboration and discussion. However, there is strong evidence to suggest that students owning their own machine, personalising it and developing their understandings over time, brings the maximum benefit in the long term.

We would welcome your feedback and suggestions, whenever you try any of these activities with your students, to help us develop future editions of this booklet. Hewlett Packard is proud of the support that it offers and you should be confident to ask for any support you may need. Manuals, guides, new downloads and details of training and support is available at www.hpgraphingcalc.org.

Chris Olley
Updated October 2013

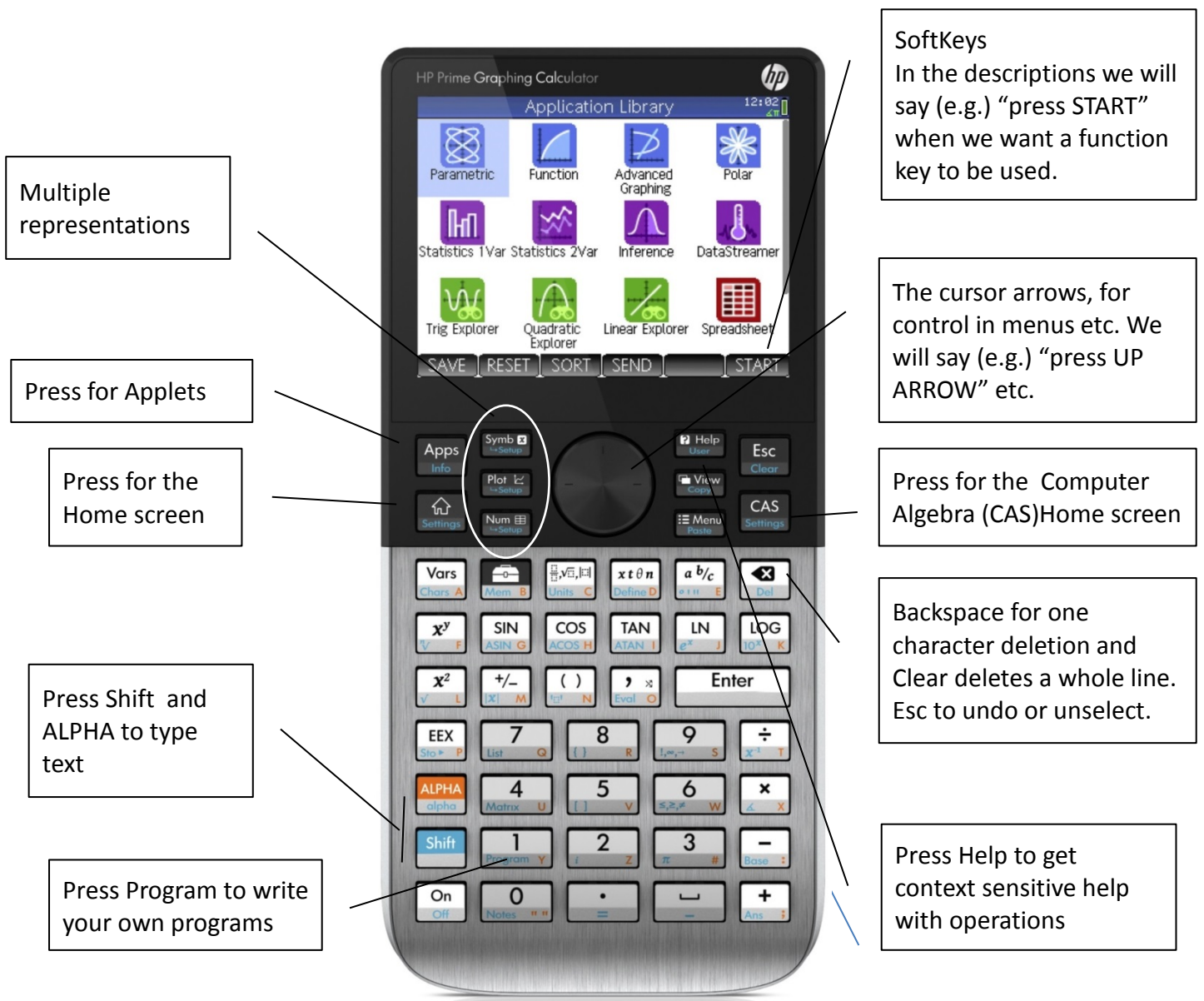
Getting Started with HP Prime

First, remember, HP Prime has a multi-touch touchscreen. Look for options and opportunities on screen to press to get started. To start calculating, press **Home**. Then, notice there is no = key. Use the **Enter** key to evaluate.

If you get stuck, press the **Home** key to get back to the home screen. Press Esc, to deselect or move back in the menus. Press the **Apps** key, to return to the opening Apps menu. Press this to find all of the built in applications. This is the central place for advanced functions and graphing.

Find the **Symb**, **Plot** and **Num** keys. These allow you to instantly switch between the three representations: symbolic, graph and table.

The bottom of the screen shows SoftKeys. Press these on screen. In the text we will refer to these by name, for example, we will say, “press START”. These are the context options (like right clicking in computer software), so look here for options.



Getting Started with HP39gII

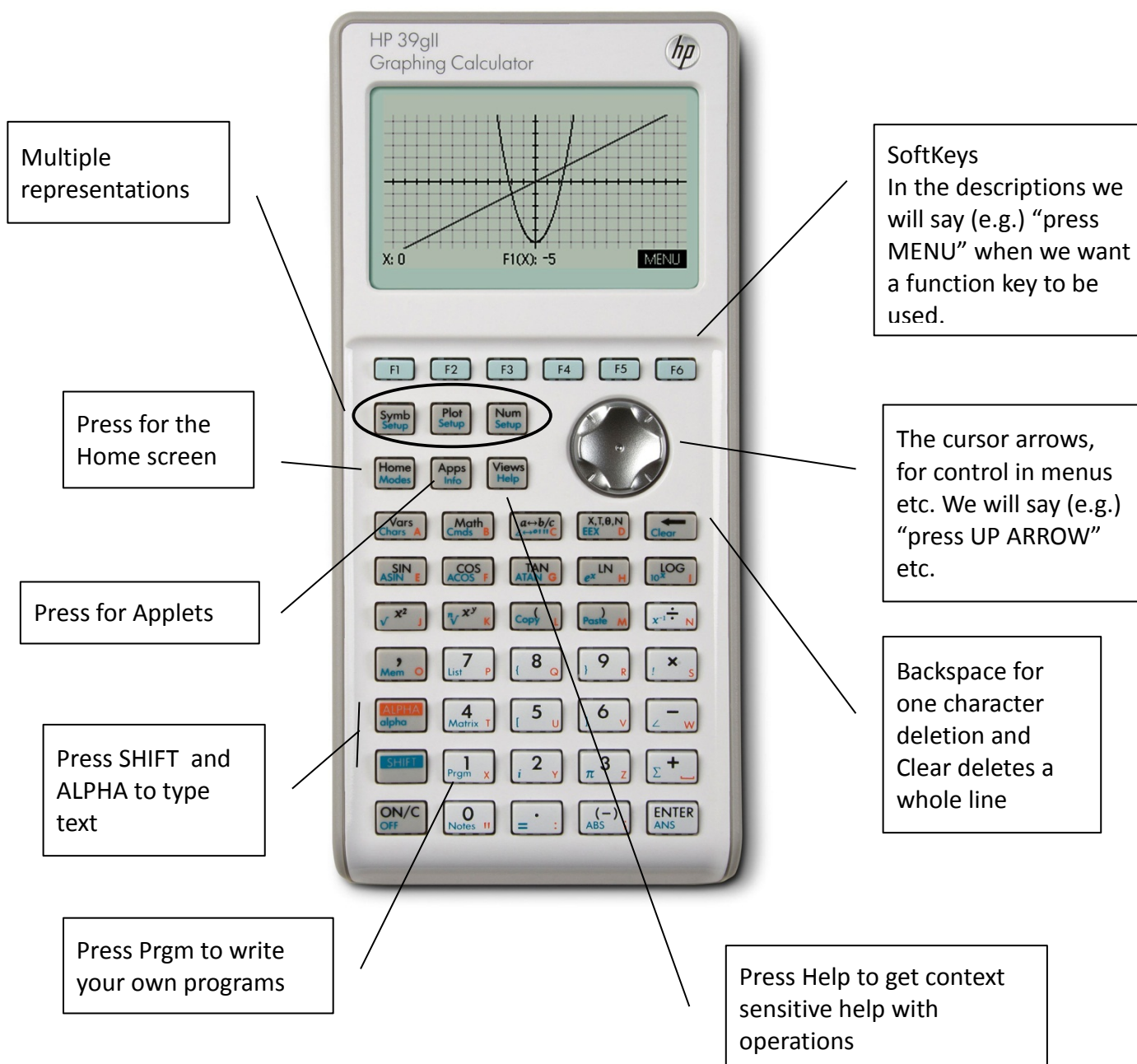
First, notice there is no = key. Use the **Enter** key to evaluate.

Now find the **Home** key. If you ever get stuck, press the home key to get back to the home screen.

Now find the **Apps** key. Press this to find all of the built in applications. This is the central place for advanced functions and graphing.

Find the **Symb**, **Plot** and **Num** keys. These allow you to instantly switch between the three representations: symbolic, graph and table.

The top row of 6 HP39gII keys are SoftKeys. Look at the bottom of the display. Small black rectangles above each SoftKey show what the SoftKey will do. In the photo, pressing the right most SoftKey will show the graph menu. On the screen shown to access the menu press F6 ... we would say “press MENU”.



1: Number Explorations

It is important to get students started with an activity which will make them comfortable with the machine. For any new group I have always started with a lesson asking students to explore the different functions that they can find. They can be guided to focus on the two rows of keys starting with SIN and X^2 and those below. I have found that if the teacher continues to prompt students simply to keep trying out new numbers and new functions, then interesting discoveries are made. Students can present their findings at the end of the lesson. In my experience, in very ordinary schools, key stage 3 students will say things like “that SIN button, doesn’t matter which number you try, the answer is always less than 1”. Some students will explore lots of features and the teacher may need to keep them focussed on one interesting idea or may be prepared to let them try out anything!



Activities for Exploring Properties of Number

A very powerful use of this technology is to quickly generate lots of data from which students can draw inference. For me, it is much better if students can work out how mathematical processes work and then report their thinking to the teacher and their peers, rather than the teacher tell them their way of doing things. The learning is then much more secure and expressed in terms the student understands because they did it themselves. I use a set of simple activities (which nonetheless can generate some very thoughtful mathematics), to help students get started with this way of working. These activities could all be done just as well on a scientific calculator, but the aim here is to develop a way of working which can then take advantage of the advanced possibilities of the graphing calculator.

Exploring decimal representations of fractions

Press: Shift Settings/Modes. Make sure the *Number Format* is set to standard. Then press Home.

Now enter fractions to explore e.g.

$1 \div 2$ Enter

$1 \div 3$ Enter

$1 \div 4$ Enter

$1 \div 5$ Enter

$1 \div 6$ Enter

etc.

Function	13:23
$\frac{1}{2}$.5
$\frac{1}{3}$.333333333333
$\frac{1}{4}$.25
$\frac{1}{5}$.2
$\frac{1}{6}$.166666666667

Sto ▶

Students can be asked to conjecture in each case. The order given is intriguing because it suggests a pattern which does not emerge. Students can look for relationships e.g. between 0.5 and 0.25 and 0.33333... and 0.1666... Important details such as the suppression of trailing zeros and final rounding ($1 \div 6 = 0.1666\dots67$) become issues for discussion. Then students can explore with the calculator to find families of fractions and relationships between their decimal representations.

Exploring addition of fractions

Use the $a \leftrightarrow b/c$ key to show the result as a fraction.

On HP Prime after entering the first fraction, press *Right* to enter the operation.

Now enter added fractions to test

- $2 \div 7 + 3 \div 7$
- $4 \div 13 + 7 \div 13$
- $1 \div 3 + 1 \div 5$
- $2 \div 3 + 1 \div 5$
- $2 \div 5 + 3 \div 7$

Function		13:29
$\frac{2}{7} + \frac{3}{7}$		$\frac{5}{7}$
$\frac{4}{13} + \frac{7}{13}$		$\frac{11}{13}$
$\frac{1}{3} + \frac{1}{5}$		$\frac{8}{15}$
$\frac{2}{3} + \frac{3}{7}$		$\frac{23}{21}$

Sto ▶

Students should write the correct form of the output into their notebooks. $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

The examples given are chosen to support students to explore each type thoroughly. They should try different examples of each type, make and then test conjectures about how the fractions are being added. They can then move on to the next type to see if their conjecture still holds or if they need to develop it further. In the end they should have a secure mechanism for adding fractions which they can explain and exemplify.

Note that the calculator will always simplify fractions, so it may be necessary to guide students to examples which allow them to investigate the effect of this.

Exploring Standard Form

Make sure the number mode is set to scientific and fixed to (say) 3 d.p.

Shift Settings/Modes Choose Scientific → 3. Then press Home

- e.g.
- 56000 Enter
 - 0.37 Enter
 - 0.00000458 Enter

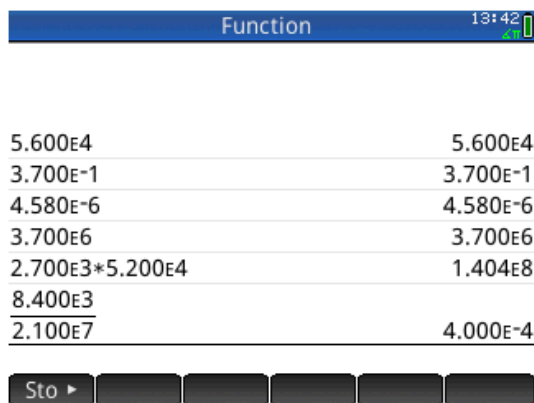
Write the results in standard form:

The calculator reports 56000 as 5.600E4

Students should write $56000 = 5.6 \times 10^4$

then: 370 shift EEX 4 Enter

Again, students should generate sufficient data to be able to describe their process and give examples in each case (convert into standard form, convert from standard form, numbers greater than 1 and numbers between 1 and -1 and finally arithmetic on numbers in standard form). As always it is important for the students to write their data, correctly formatted into notebooks. This allows a clear focus on accurate written mathematics.



Exploring Factors

By generating lots of data from prime factorisations students will develop an idea of prime numbers as those with exactly two distinct factors and how only these numbers can be used to make up all other numbers. They can develop this work by investigating the effect of the Factor command on algebraic expressions.

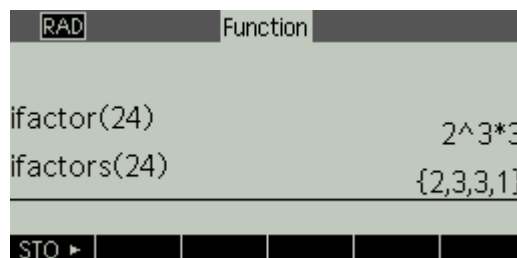
The objective is to develop an understanding of prime numbers, methods for prime decomposition and the process of factorisation in general.

We use the ifactor (for integer factorisation) command in the Math menu.

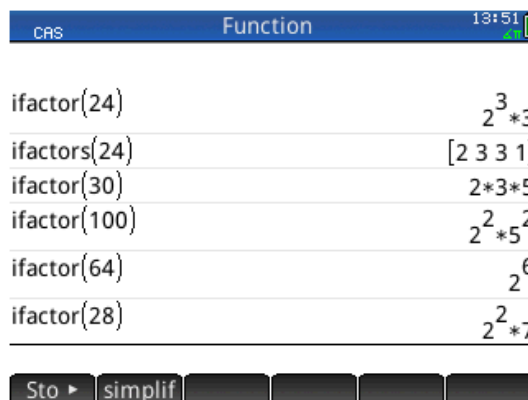
On HP39gII:

Press Math and navigate to Integer then ifactor type a number, say 24 and close the bracket. Press Enter.

You can also see a list of the factors using the ifactors command.



On HP Prime, the ifactor command is a CAS command. You can use this in the Home screen, but it is better in the CAS screen. So, press the CAS button. The press the Suitcase button and choose Integer/Factors (in the CAS tab). The list of factors is available with Factor List in the same menu.



This yields the result $24 = 2^3 \times 3$

2: Algebra Explorations

Exploring algebraic expressions

[Note **Sto** is the left SoftKey on the home screen]

The idea here is to get students to experiment with the values of expressions. They choose values for (say) 3 different letter variables.

Set some values for A, B, and C e.g.

5 STO ALPHA A Enter

2 STO ALPHA B Enter

+/- or (-) 1 STO ALPHA C Enter

Students can then type different expressions to test. This works well as a paired activity. Students are allowed a maximum of (say) 5 characters to type an expression. Their neighbour has to say what the value of the expression is, before the first student presses the Enter key to evaluate.

Function		10:24
5▶A		5
2▶B		2
-1▶C		-1
5*A+4*B		33
(A+B) ²		49
2*C ²		2
A+C		
B		2

Sto ▶

Now test different expressions (for example)

5 ALPHA A + 4 ALPHA B Enter

(ALPHA A + ALPHA B) X² Enter

2 ALPHA C X² Enter

ALPHA B ^ ALPHA C Enter

(ALPHA A + ALPHA C) ÷ ALPHA B Enter

(HP39gII) To show the algebra correctly SHIFT/Modes/Page down/Textbook display

RAD	Function
(A+C)	
B	2

Sto ▶

Students should copy examples into their notebooks.

Place Value and the Algebraic Symbol System

This activity gets students to explore place value by setting up expressions. First, the letter variables A to F are defined as a suitable set of powers of 10.

Press 1000 Sto ALPHA A Enter
 Press 100 Sto ALPHA B Enter
 Press 10 Sto ALPHA C Enter
 Press 1 Sto ALPHA D Enter
 Press .1 Sto ALPHA E Enter
 Press .01 Sto ALPHA F Enter

A is 1000
 B is 100
 C is 10
 D is 1
 E is 0.1
 F is 0.01

We suggest that the teacher does this on the board before students have access to the calculators themselves. They can then have a worksheet to fill in like the one on the next page. Students can be allocated a calculator when they have filled in the 'My Idea' column for every question. They then follow the instructions to set up the calculator and check, filling the calculator response into the end column.

After this students working in pairs can set and test challenges for each other as before.

Function		10:27
1000►A		1000
100►B		100
10►C		10
1►D		1
.1►E		.1
.01►F		.01
3*A+5*C		3050
2*B+E		200.1

Sto ►

Function		10:27
1►D		1
0.1►E		0.1
0.01►F		0.01

STO ►

Function		10:27
1000►A		1000
100►B		100
10►C		10

STO ►

Function		10:27
3*A+5*C		3050
2*B+E		200.1

STO ►

Place Value and the Algebraic Symbol System Worksheet

Press 1000 Sto ALPHA A Enter
 Press 100 Sto ALPHA B Enter
 Press 10 Sto ALPHA C Enter
 Press 1 Sto ALPHA D Enter
 Press .1 Sto ALPHA E Enter
 Press .01 Sto ALPHA F Enter

A is 1000
 B is 100
 C is 10
 D is 1
 E is 0.1
 F is 0.01

1.		My Idea	Calculator check
	3A		
	5C		
	2B		
	8E		
	A + B		
	2B + C		
	5C + D		

2.		My Idea	Calculator check
	D + E		
	3C+2E		
	A+F		
	B+2C+3D		
	5B+D+3E		
	6A+3D+F		
	3C+D+E+7F		
	6A+E+F		

3.		My Idea	Calculator check
	700		
	0.8		
	15		
	5020		
	307.6		
	5004		
	12.83		
	5070.09		

3: Exploring Calculus with CAS (HP Prime only)

Having a machine which can find do the algebra for you is very powerful. Clearly, if an exercise question (or exam) requires a student to just do the algebra, then the machine cannot be used. So, we have two principal opportunities:

- Use the calculator to provide plentiful data to explore how the algebra is being done.
- Support more complex tasks by giving confidence in the algebra, to free up thinking time on the mathematics.

We will be using the CAS functions in this section. So, on your HP Prime press the CAS key to open the CAS home screen.

Start by exploring the sums of series. Use the template key to insert a summation template.

Use the left and right cursor keys to navigate to each entry point in turn, to input a suitable sum. Use ALPHA to choose your variable. (Notice that CAS variables are always lower case, so HP Prime defaults to lower case when ALPHA is used in the CAS screen).

Press enter to generate the solution (55)

CAS Function 10:33

$$\sum_{x=1}^{10} (x)$$

55

Sto ► simplif

Now repeat to find the general expression for the sum to n .

Unsimplified solutions are always displayed first, so keep pressing **simplif** to simplify further.

CAS Function 10:36

$$\sum_{x=1}^n (x)$$

$$\frac{(n+1)^2 + n - 1}{2}$$

simplify $\left(\frac{(n+1)^2 + n - 1}{2} \right)$

$$\frac{n^2 + n}{2}$$

Sto ► simplif

Press up to highlight a past entry, then Copy. Now make some change to the expression and see what difference it makes. Form conjectures and test them.

CAS Function 10:39

$$\sum_{x=1}^n (x^2)$$


$$\frac{2*(n+1)^3 - 3*(n+1)^2 + n + 1}{6}$$

simplify $\left(\frac{2*(n+1)^3 - 3*(n+1)^2 + n + 1}{6} \right)$

$$\frac{2*n^3 + 3*n^2 + n}{6}$$

Sto ► simplif

You will also have seen calculus templates available from the template key.

On the numeric home screen  we can work numerically. e.g. choose the d/dx template, Enter X as the variable and X² as the function. It will evaluate the differential at the current value of X. Use Sto to change the value of X. In this screen I have used Sto to set X=4 then entered the differential.

Use the integration template for numeric integrals, for example Enter: $\int_1^5 x^2 dx$ and it will be evaluated using the current value of X. (Here it is still X=4 entered earlier.

Press CAS to return to the CAS home screen and explore the calculus commands again.

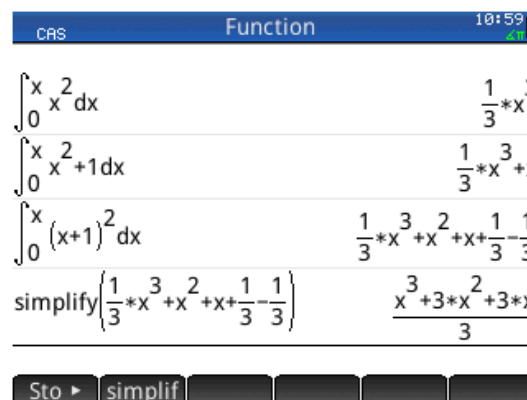
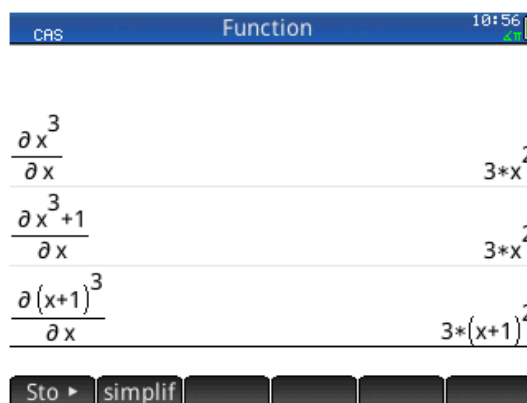
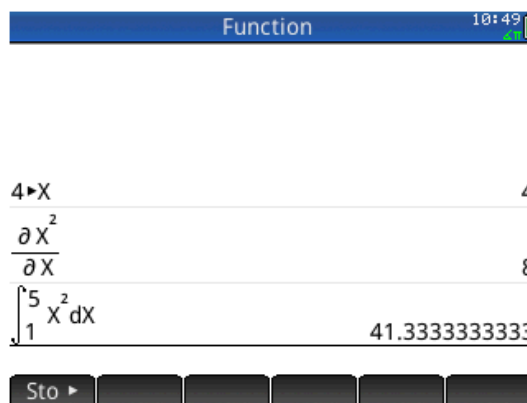
Using the d/dx template, type in suitable elements using the cursor. Then press Enter.

(Press simplif to simplify if needs be).

Students can now explore how the sums, differentials and integrals change with changes of function and with changes of values and limits. They can categorise calculus methods.

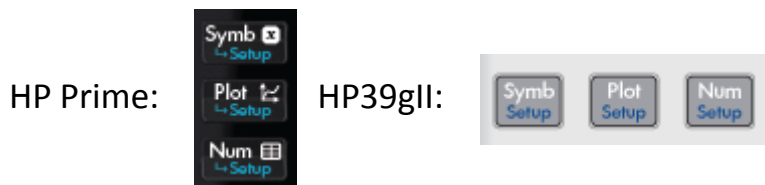
How open or how structured their exploration, is down to the teacher's judgement, but the calculator provides any amount of data needed.

Here is an example of integration. Experiment with different settings for the limits, to handle the constant of integration.



4: Exploring Graphs

The graphing calculator naturally offers very quick and easy to use graphing software. Most graphing is controlled from the top row of labelled keys. These give quick access to the *multiple representations* of functions: algebra, graph, table. These are the SYMB, PLOT and NUM keys.



Notice that Setup options are available for each view by pressing shift plus the appropriate key.

You will want to put a variable in when you define a function. You can use the ALPHA key to choose any letter. However, there is a x t θ n (HP Prime) or X,T, θ ,N (HP39gII) key, which chooses the correct variable for any given graph type. Also, an on-screen SoftKey is available for the standard variable.

By default, the calculator graphs function plots $y = f(x)$. (You will see 'Function' showing in the top line of the screen, to show that the Function App is running). However, there are many other types of function which can be plotted. Press the Apps key to see what is available:

- Function
- Parametric
- Polar
- Sequence

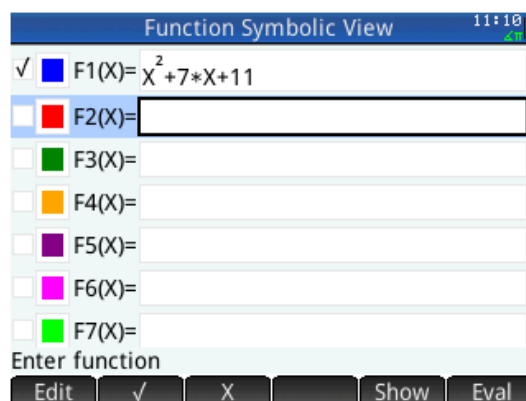
You can store a number of functions of each type and keep them in the calculator's memory, switching between the different types using the Apps key. We suggest that whenever you start a new activity, you RESET the applet before starting. This way all of the settings will be returned to normal and any graphs from an earlier user will be deleted.

Try this out to get started:

Press Apps navigate to Function and press Reset then OK then Start

[This achieves the same effect as pressing the Symb button on a new calculator]

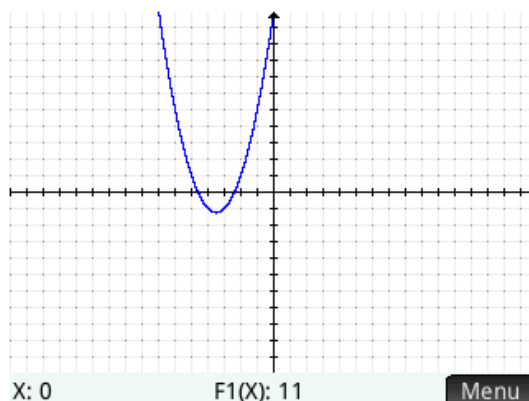
Type a function (say) $X^2+7X+11$ and OK



Press the Plot key.

Notice the SoftKey Menu. This gives access to all of the graphing facilities.

- You can Zoom in on parts of the graph.
- You can Trace to see values of points on the graph.
- The Fcn key allows roots, turning points, slope, etc. to be found.



A fast zoom is available by pressing the + and – keys. This zooms in and out using the current position of the cursor as the centre. Use this with care!

Press the Num key.

You can navigate up and down the table to look for (say) turning points or points of intersection.

You can even zoom in the table, to find (say) roots to greater levels of accuracy.

Function Numeric View			
X	F1		
0	11		
.1	11.71		
.2	12.44		
.3	13.19		
.4	13.96		
.5	14.75		
.6	15.56		
.7	16.39		
.8	17.24		
.9	18.11		
0			

Buttons: Zoom, Size, Defn, Column

Polar Graphs

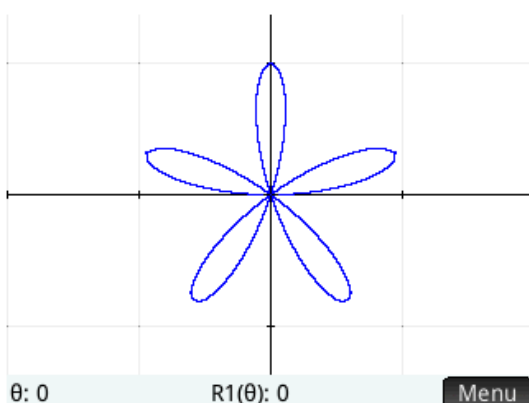
Now press Apps, navigate to Polar and press Reset then OK then Start.

Notice the function definitions are now in polar form. Type $\sin(5\theta)$. The $x t \theta n$ key now inserts a θ instead of an X. Press OK.

Press PLOT to see a tiny graph.

Press the + or – keys to get a good view.

Press Apps, navigate to Function and press Start (**Do not** press Reset!) You will see that your previous graph is still available as you left it.



Sequence Graphs

Press Apps, navigate to Sequence and press Reset then OK then Start.

You can enter the first number in the sequence $U1(N)$ and optionally, a second number in the sequence $U2(N)$ (useful notably to enter a Fibonacci sequence). Then enter a rule for the sequence, either in term-to-term or position-to-term form. Notice that when you move to the $U1(N)$ line the on-screen SoftKeys give the building blocks required.

Here, we have given a simple sequence starting at 2 and with each term being the previous term plus 3.

Pressing Num shows the sequence.

Pressing Plot gives a rather different view, which provides new materials for comparisons and a clear route to discussing the move from discrete to continuous functions.

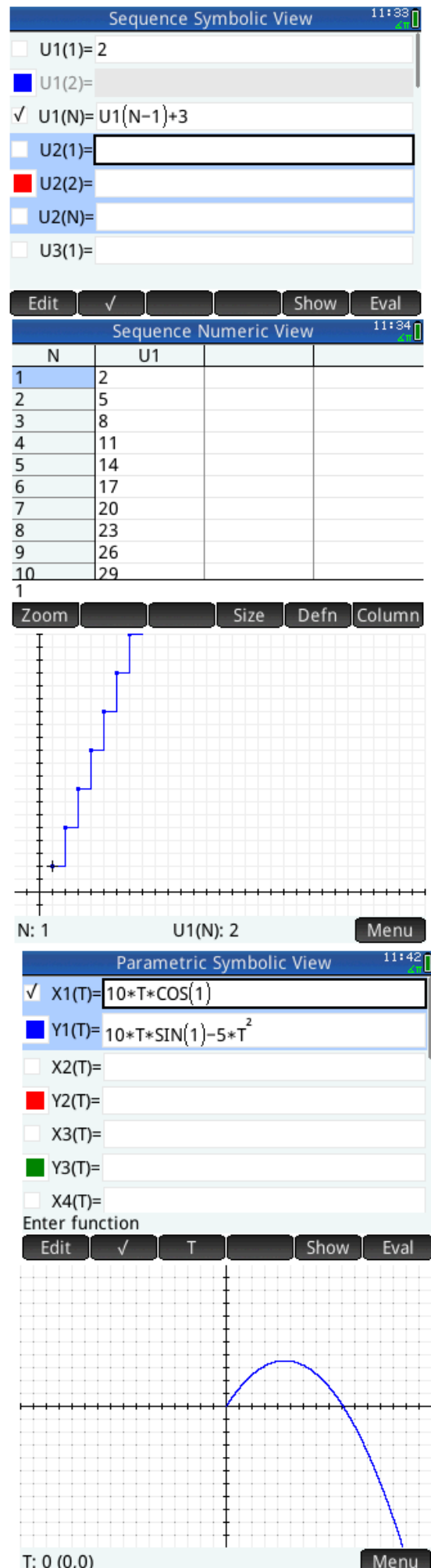
Parametric Graphs

Here is a classic example of a parametric graph:

You should experiment with all of the different graph and function types. Try out different Zoom's. See what Setup options are available in each applet and in each representation. Your students now have an extremely flexible tool to explore functions of almost any type choosing the most suitable representation

to solve the problem at hand.

The next two pages contain a worksheet you can use which structures a student centred investigation into the effect of changing the coefficients on the graph of a quadratic function.



Investigating Quadratics Worksheet

A Quadratic Equation has the form

$$ax^2 + bx + c$$

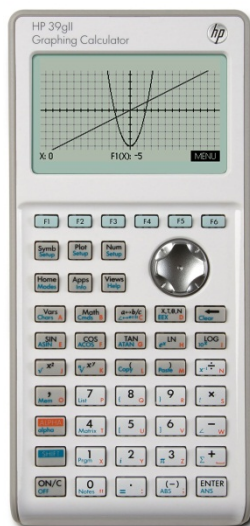
These are all quadratic equations

$$y = x^2$$

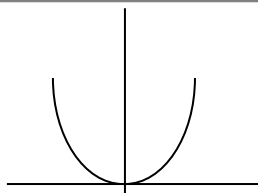
$$y = x^2 + 3x$$

$$y = x^2 - 4$$

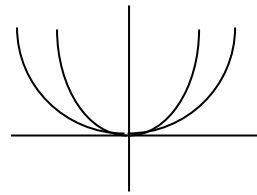
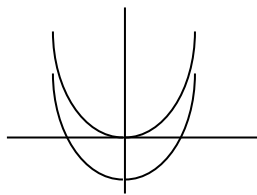
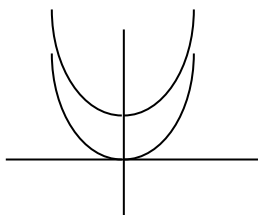
$$y = x^2 + 2x + 1$$



1. Graph $y = x^2$

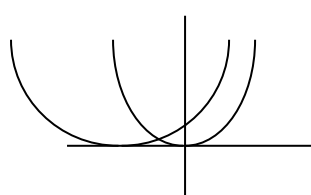
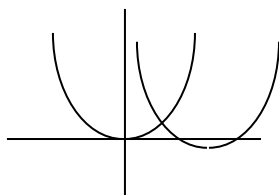
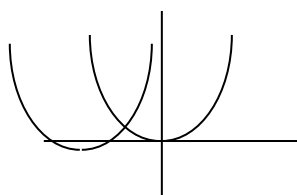


2. Make graphs similar to these.
Write down the equations you have used.



Make as many pairs as you can for each graph.

3. Make graphs similar to these.
Write down the equations you have used.



Make as many pairs as you can for each graph.

4. Write notes to explain how a graph of $y = x^2$ can be changed:
- to move up or down the y axis.
 - to move left or right on the x axis.
 - to get wider or thinner.

Hint: Try $y = (x - 1)^2$

5. Draw graphs of $y = x^2 + 2x$ and $y = x^2 - 4x$.

Draw more graphs, changing only the coefficient of x .

Explain the effect of changing the coefficient of x .

Note: the number that we multiply the x by is the called the *coefficient* of x .

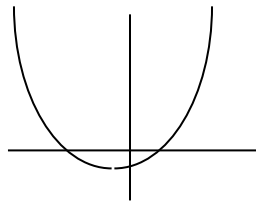
6. We can write a quadratic equation like this: $y = (x - 3)(x + 5)$

Multiply out the brackets and write down the expanded form of the equation.

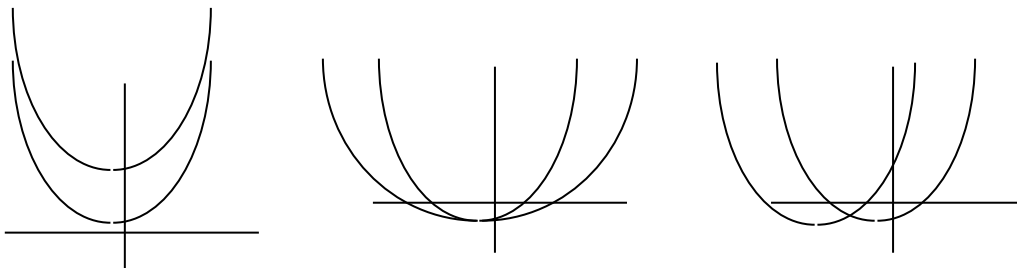
Try out equations of this type. Change the numbers. Change the $+$ and $-$.

Explain what effect the numbers 3 and 5 have on the position of the graph.

7. Draw a graph of $y = (x - 3)(x + 5)$



Make graphs similar to these. Write down the equations you have used.



Make as many pairs as you can for each graph.

8. Write notes to explain how a graph of $y = (x - 3)(x + 5)$ can be changed:
- (a) to move up or down the y axis.
 - (b) to move left or right on the x axis.
 - (c) to get wider or thinner.

Quadratic Roots

Try this activity yourself to consider the advantages of different methods for finding roots of a quadratic.

Method 1

Press Apps, navigate to Function and press Reset then OK then Start

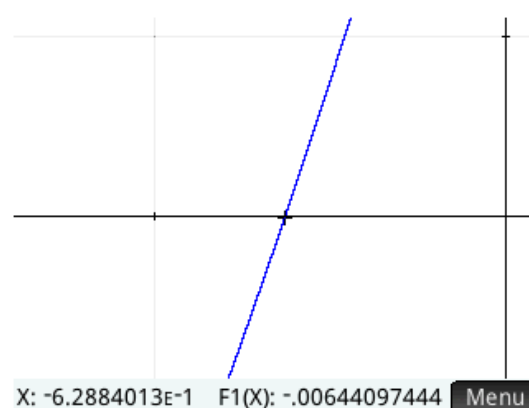
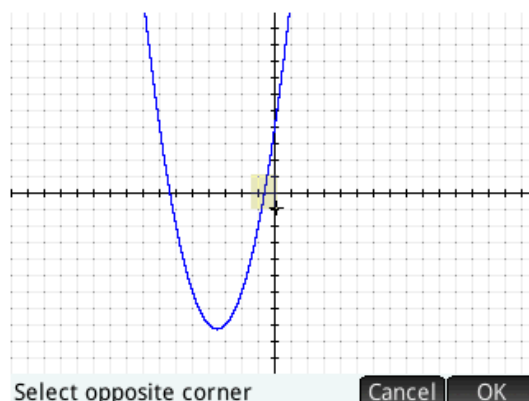
Type $X^2 + 7X + 4$ then press OK (You can use X,T,θ,N for the X then X^2)

Press Plot then Menu then Zoom.

Choose **Box...** then OK

Press on the screen (HP Prime) or move the cursor so that it a little to the left and a little above one of the roots. Press OK.

Now press again or move the cursor a little to the right and a little below the root. You will see a box formed around the root. You should aim to make a very small box with the root in the middle. If it all goes wrong you can press Zoom again and choose Un-zoom. Now press OK to complete the zoom.



Now you can press **Trace** and move the cursor as close as possible to the root, giving its value. Use Un-zoom to go back to the original graph and find the second root.

Method 2

Now find the solutions from a table of values for this function.

- Press Num and use the up/down cursor to explore the table of values.
- Find which two numbers one of the solutions of $x^2 + 4x + 7 = 0$ falls between.
- Press Shift Num and you can change the intervals in the table.
- Select Num Step and type 0.01 then OK then press the Num key.
- Repeat this to find both solutions correct to 2 decimal places.

Method 3

Now get the calculator to find the roots.

- Press PLOT MENU FCN ROOTS
- This will show the root closest to the cursor
- Move the cursor with the arrow keys closer to the other root
- Press MENU FCN ROOTS
- This will show the other root

5: Guess the Function

This activity is ideal to get students to feel the variability in the function. It is really important to avoid prompting but to leave students make guesses and get them wrong! As the amount of data increases, they will get better in their ability to see the outcomes. The teacher sets up a function unseen and students guess the output for different inputs. Students are in complete control of the inputs, but the teacher gradually helps them by increasing the views they can see. First, the values of x can be sorted to be in order. It is worth waiting for a view values to be considered before doing this, so students are aware of the power of this simple change. Critically, students can be shown a graph. Here, we set the calculator up to avoid showing the axes. In this way, students focus on the shape only. Again, the graph should only be shown after a lot of data has been considered. I show it only for a couple of seconds and then return to the table of values. The intention is to help get a 'feel' for the function. Then with all this input students will be able to give inputs and outputs, before the function can finally be revealed.

This is how to do it:

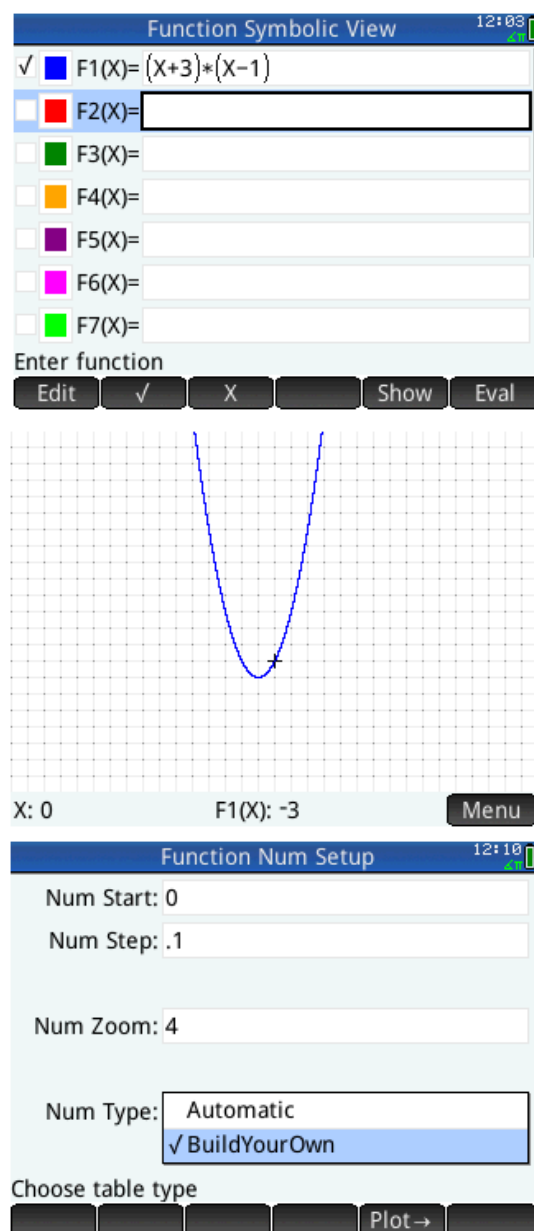
First press Apps, choose Function, Reset, OK and Start. Enter a function for your students to guess. (I find a quadratic in the form $(x + a)(x + b)$ works well).

Now click Shift and Plot to enter the plot setup. Click the Page 1/2 softkey, select Axes and click the \checkmark /CHK softkey to deselect the axes. Click Plot to make sure you can see the graph properly (you may need to use Menu then Zoom to set it up, but quicker to choose a function whose graph you can see!)

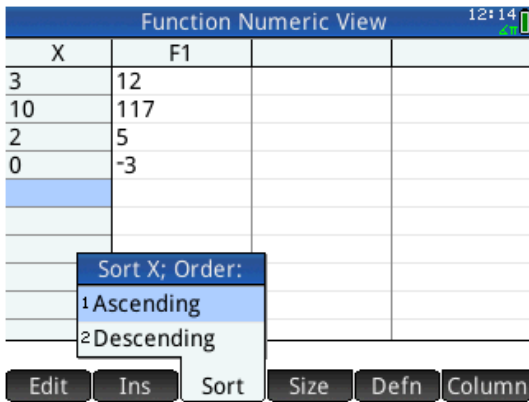
Now click Shift Num to enter the number setup, select Num Type, click the Choose softkey, select BuildYourOwn (and click OK).

Now click Num and you are ready to start.

Ask participants to give a value for X and guess the value for $F1$. Make sure no-one actually guesses the function or that will close down everyone else's thinking.



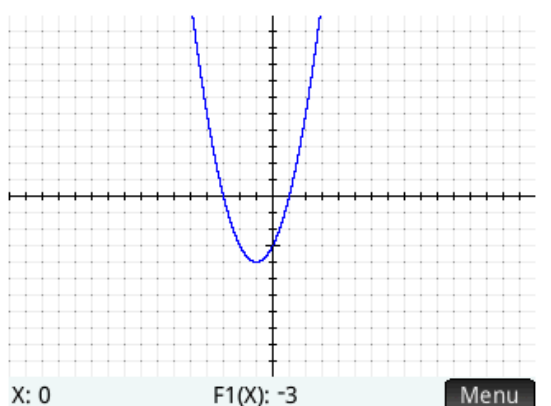
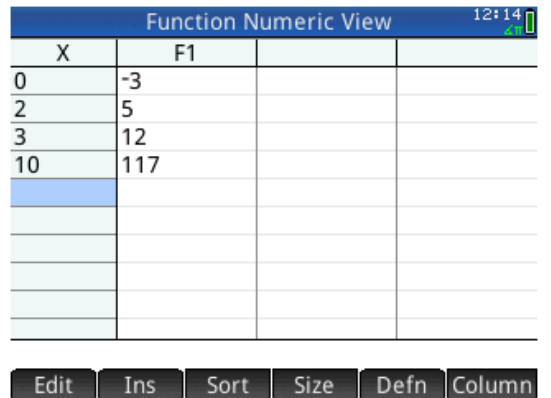
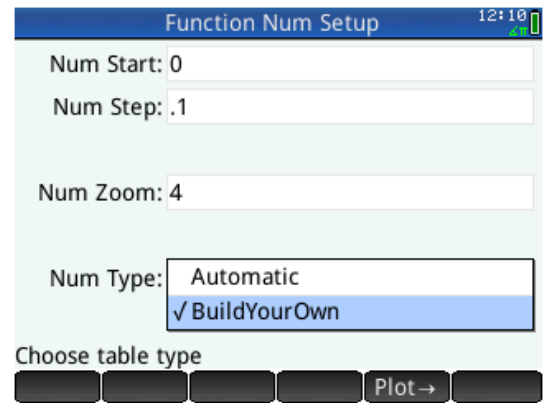
When you have a few values, ask if we could improve the table and they will say (because they always do!) “put them in order”, so click the SORT softkey.



Only when there is enough data and some are struggling, ask if another view would be useful and they’ll say “show us a graph”. So, click Plot, but only very briefly, click Num again after a couple of seconds. They only need the shape after all!

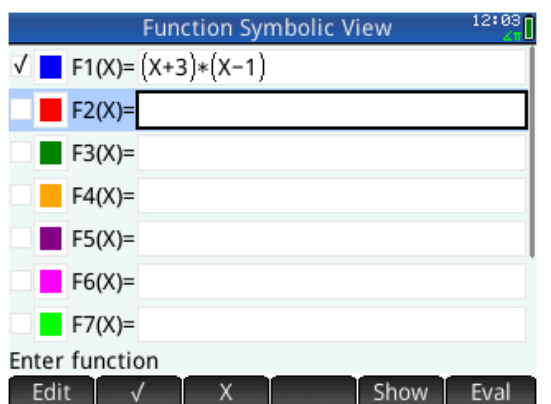
Now let participants try more new values until they are all confident they have found the function.

As a final view, you can turn the axes back on (Shift/Plot Setup/Page 1/2 /Axes/✓ or CHK)



When most participants are able to give a value for X and its corresponding value of F1, ask them what form they think you put the function in. Finally, the function can be revealed.

It is interesting to discuss how different algebraic forms are better or worse for identifying the graph.



6: Pythagorean Triples

This activity is based on an article by Bill Richardson which is highly recommended and will provide all you need to extend this introduction (last accessed 18.10.13): www.math.wichita.edu/~richardson/pythagoreantriples.html

The idea is that if we know the first 4 Pythagorean triples in a sequence connected by some rule, then we can find a generating matrix to produce the others. Students can generate triples using a spreadsheet to find sets in this way. The most well-known would be where the last two values are consecutive integers. Now, students could find an algebraic generator for this set, but this is very hard for some other sets, so see how the matrix method works.

The first 4 triples are:

[3 4 5], [5 12 13], [7 24 25], [9 40 41]

Now we can set up a matrix statement showing the first three triples being generated by the 2nd, 3rd and 4th.

$$\begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

Which we can solve by pre-multiplying by the first matrix i.e.

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 3 & 4 & 5 \\ 5 & 12 & 13 \\ 7 & 24 & 25 \end{pmatrix}^{-1} \begin{pmatrix} 5 & 12 & 13 \\ 7 & 24 & 25 \\ 9 & 40 & 41 \end{pmatrix}$$

On the calculator we can find the generator matrix using the MATRIX facilities. First we create the two matrices:

Press Shift and Matrix. The cursor should be on the top row for M1. Now press Edit. Enter the values one at a time pressing OK. For the third entry you will need to use the cursor to move to the end of the first row. Press OK. It will now carry on, knowing that the matrix has 3 columns.

The matrix M1 is now defined, so press MATRIX again and repeat for M2

The screenshot shows the 'Matrices' screen on an HP Prime calculator. The title bar says 'Matrices' and the time is 12:27. The matrix M1 is being defined with 3 rows and 3 columns. The values entered are:

M1	1	2	3
1	3	4	5
2	5	12	13
3	7	24	25

Ins Size Go → Column

The screenshot shows the 'Matrices' screen on an HP Prime calculator. The title bar says 'Matrices' and the time is 12:29. The matrix M2 is being defined with 3 rows and 3 columns. The values entered are:

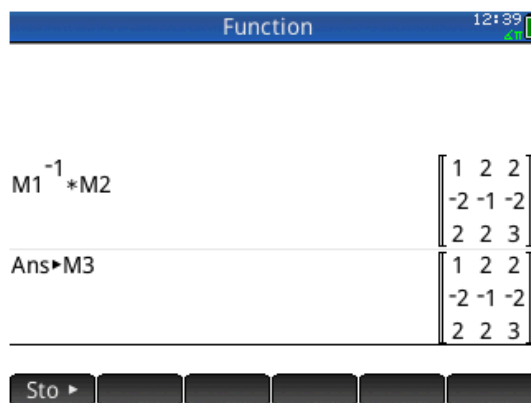
M2	1	2	3
1	5	12	13
2	7	24	25
3	9	40	41

Ins Size Go → Column

We can now find the generator matrix. So, press HOME and type the statement $M1^{-1} * M2$ and press Enter.

To store this as a new matrix, press Sto then type M3 and press Enter.

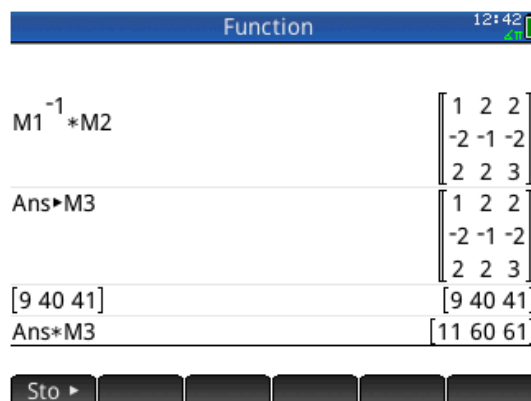
Note: To enter matrices directly on the home screen we use the square bracket, one bracket for the matrix, then another for each new row. Everything separated by commas.



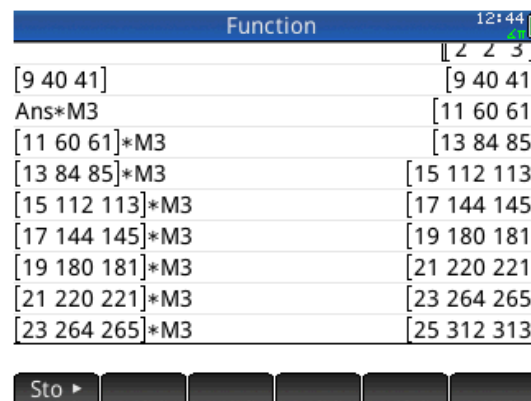
We can now generate triples:
(HP 39gII) Enter the last triple we know as $[[9,40,41]]$

(HP Prime) Choose square brackets and enter 9 right 40 right 41 Enter

Now press \times M3 and Enter
The next triple is $[11,60,61]$



Now use up arrow and COPY then \times M3, then Enter to find the next, and the next, ...



I hope this is sufficiently compelling an idea for students to want to work out how it works. The powerful matrix tools make the mathematics possible. Students need now to go away and work out how matrix multiplication and matrix inverses work. Then they can use this knowledge to work out how the method works and hopefully go on and prove it.

To practice, here are two further sets of triples. Firstly, where the first and last numbers are consecutive odd numbers:

$[3 \ 4 \ 5]$, $[15 \ 8 \ 17]$, $[35 \ 12 \ 37]$, $[63 \ 16 \ 65]$

Secondly, where the first two numbers are consecutive whole numbers:

$[3 \ 4 \ 5]$, $[20 \ 21 \ 29]$, $[119 \ 120 \ 169]$, $[696 \ 697 \ 985]$

7: Statistics

A very powerful feature of the graphing calculator is the ability to generate statistical charts and graphs and summary statistics for data sets, very quickly. This allows students to see the multiple representations: the data set, the chart, the summary statistics. They can see the relationships between these representations. If we have single variable data, then we are likely to be comparing two or more data sets. We want to see the differences in distribution and we want to find summary statistics to make comparisons. Box-and-whisker plots are widely seen to be excellent visual tools for comparing single variable data sets. They clearly show the inter-quartile range together with the outlying quartiles and visually showing the average (median), and the range and inter-quartile range. Two such plots side by side allow powerful and immediate inference to be drawn. The HP Prime and HP39gII allow up to three box-and-whisker plots to be drawn side-by-side.

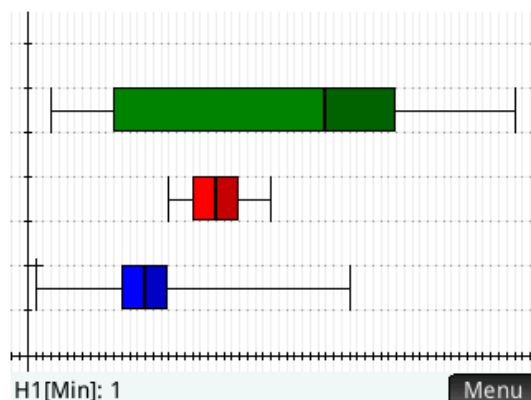
Step-by-step instructions for this work are on the following pages.

Students could collect data on (say) the age profile of visitors to three restaurants in town to support an investigation into their marketing requirements.

We can enter the data, draw the plots and see detailed summary statistics:

Statistics 1Var Numeric View 15:54				
	D1	D2	D3	D4
1	3	18	3	
2	15	21	28	
3	14	23	45	
4	12	22	5	
5	22	26	11	
6	34	31	34	
7	16	30	42	
8	1	28	58	
9	13	19	37	
10	4	19	41	
3				

Edit Ins Sort Size Make Stats



This is achieved very quickly and now allows a detailed statistical analysis of the data, to support the project at hand.

Similarly 2 variable data can be handled equally efficiently, generating scatter plots, with regression and correlations calculated and different regression analysis available.

Step-by-step instructions start on page 27.

Stats 15:57			
X	H1	H2	H3
n	30	19	27
Min	1	18	3
Q1	12	21	11
Med	15	24	38
Q3	18	27	47
Max	41	31	62
ΣX	465	459	913
ΣX^2	9301	11357	39513
\bar{x}	15.5	2.415789E1	3.3814815E1
sX	8.49644956	3.86239934	1.8229387E1
30			

Size Column OK

Drawing Three Side-by-Side Box and Whisker Plots

Choose the Statistics Applet:

Press Apps and use the arrow keys to find Statistics 1Var

Use the SoftKeys for Reset and OK and Start

Enter your Data:

Type data in column D1 and press Enter repeat until finished

(Use the right and left arrow keys to move between the lists)

- Under D1 enter the data for the first set
- Under D2 enter the data for the second set
- Under D3 enter the data for the second set

Choose the Plots:

Press the Symb key

Plot H1 will already be set to D1

Press the down arrow to set plot H2 use the SoftKey then 2 then OK

Repeat for H3.

For each plot select the Plot type (a drop down on HP Prime) and press CHOOS (HP39gII) and select BoxWhisker and OK

Now press the Plot key and press Menu/Zoom/Autoscale to optimise the view.

Get the Statistics:

In the Plot view, press Menu/Trace and use the cursor keys to show on-screen summary statistics.

- Minimum value
- Lower Quartile (Q1)
- Median
- Upper Quartile (Q3)
- Maximum value

Students can work out and write down the interquartile range.

Remembering: Interquartile range = upper quartile – lower quartile

Find the Statistics for the Data

Press the Num key and press Stats for detailed summary statistics.

Press SoftKey OK when finished.

Two Variable Statistics: Correlation and Regression

Choose the Statistics Applet:

Press Apps and use the arrow keys to find Statistics 2VAR

Use the SoftKeys for Reset and OK and Start

Enter your Data:

Type data in column C1 and press Enter repeat until finished

(Use the right and left arrow keys to move between the lists)

- Under C1 enter the data for the first set
- Under C2 enter the data for the second set

Make sure both lists are the same length.

Choose the Plots:

Press the Symb key

Plot S1 should already be set to lists C1 and C2

Now press the Plot key

Press Menu Zoom and use the arrow keys to choose Autoscale

Press Menu Fit to show the line of best fit. Press again to remove it.

Find the Statistics for the Data:

Press the Num key and SoftKey Stats to show detailed summary statistics.

Press SoftKey OK when finished.

(HP39gII) Use the down arrow to see all the statistics.

Press the Symb key and the regression line will now be showing as Fit1

Press SoftKey Show to see it separately.

Statistics 2Var Numeric View 16:10				
	C1	C2	C3	C4
1	26	13		
2	21	9.5		
3	19	5		
4	23	7.5		
5	26	11		
6	21	10		
7	24	9.5		
8	23	10.5		
9	20	7.5		
10	19.5	7		

Edit Ins Sort Size Make Stats

Statistics 2Var Symbolic View 16:11

√ S1: C1 C2

Type1: Linear

Fit1: M*X+B

S2:

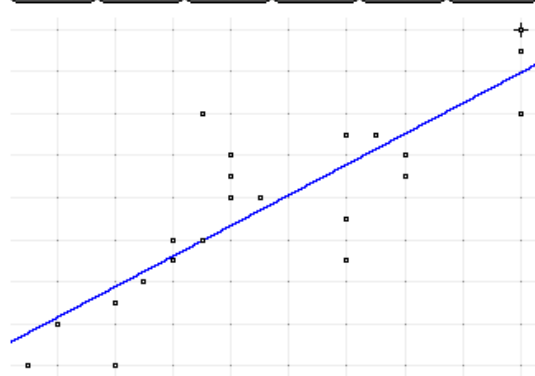
Type2: Linear

Fit2: M*X+B

S3:

Enter independent column

Edit √ C Fit Show Eval



Zoom Trace* Go To Fit* Defn Menu

Stats 16:14			
X	S1		
n	23		
r	8.459273E-1		
R ²	7.155930E-1		
sCOV	4.63537549		
σCOV	4.43383743		
ΣXY	4451.75		

23

Stats* X Y Size Column OK

Statistics 2Var Symbolic View 16:15

√ S1: C1 C2

Type1: Linear

Fit1: .729322139303*X-6.98297574627

S2:

Type2: Linear

Fit2: M*X+B

S3:

Enter independent column

Edit √ C Fit* Show Eval

8: Data Streaming

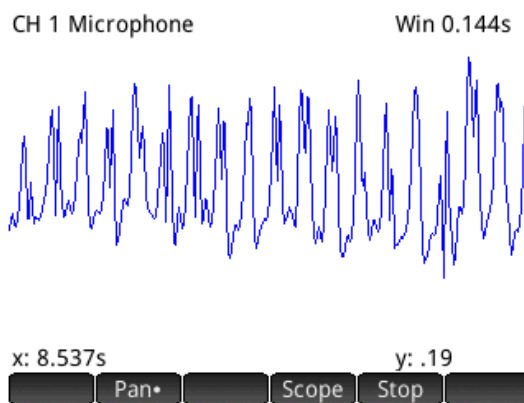
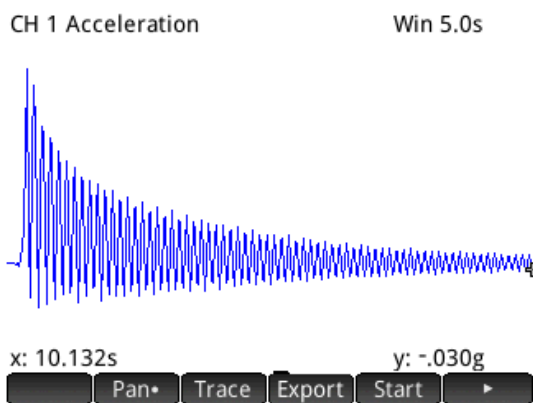
Setting Up

You will need an HP Prime or an HP39gII and a StreamSmart 410 interface unit.

You will also need suitable probes for the experiment.

Check that the probe you are using is connected to the CH1 input on the StreamSmart unit and that this is connected via it's attached lead to the calculator. Also check that the StreamSmart unit is switched on and the green light is showing that the battery is OK.

On the calculator, press Apps and use the arrow keys to find DataStreamer. When it is highlighted press the SoftKeys Reset then OK then Start.



There are a variety of probes which can be used. The first example above show the output when an acceleration probe is attached to end of a steel rule held firmly on the edge of a table and 'twanged'. The second example is the output from blowing into a microphone.

A Distance Probe can be used for matching distance time graphs. The following activity uses this feature. Also, mechanics experiments can be measured or pendula for simple harmonic motion etc. A temperature probe can be used for analysing cooling (or heating) curves.

Walking the Line

Follow the set up instructions above with the Distance Probe attached to the StreamSmart unit.

In this activity, students will walk along a line closer to and further away from the distance probe.

Put the probe on a table, facing a space at least 1

metre wide and 3 metres long. The centre of a classroom is good if the desks are moved apart a little. Find a volunteer and ask them to stand facing the probe, about 2 metres away.

We want to set up the screen so that we can see the motion.

Press the SoftKey Pan and it will change to Zoom. Press it again and it changes back. We will use Pan and Zoom to set up the screen. Pan moves the image up and down. Zoom makes the image expand in scale.

Choose Zoom and press the up arrow about three times to increase the scale.

Then choose Pan and put your hand in front of the sensor to measure near to zero metres. Now use the up arrow to move the line to just above the numbers on screen. Take your hand away to measure the person 2 metres away again.

So the 2 metre point is about 2/3 of the way up the screen (so full screen will show 3 metres) and the bottom of the screen shows zero metres (i.e. with the hand in front).

This is a little fiddly the first time, but you will get better with practice.

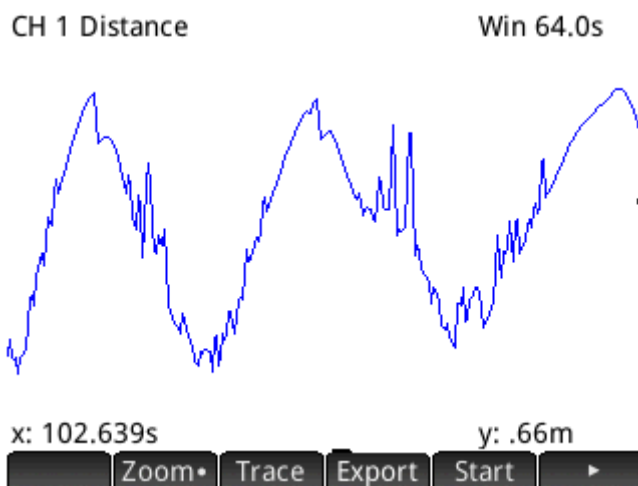
The Activity

You will need a collection of drawings of standard distance time graphs from a text book. The aim is for students to match their motion to the graph.

The student on the line should move backwards and forwards (always facing the probe) until they have created the required graph.

Teachers can be inventive with which graphs they use!

The author has prepared a presentation sequence with a variety of graphs which can be printed or displayed to support this activity. It is available on request.



Cooling Curves

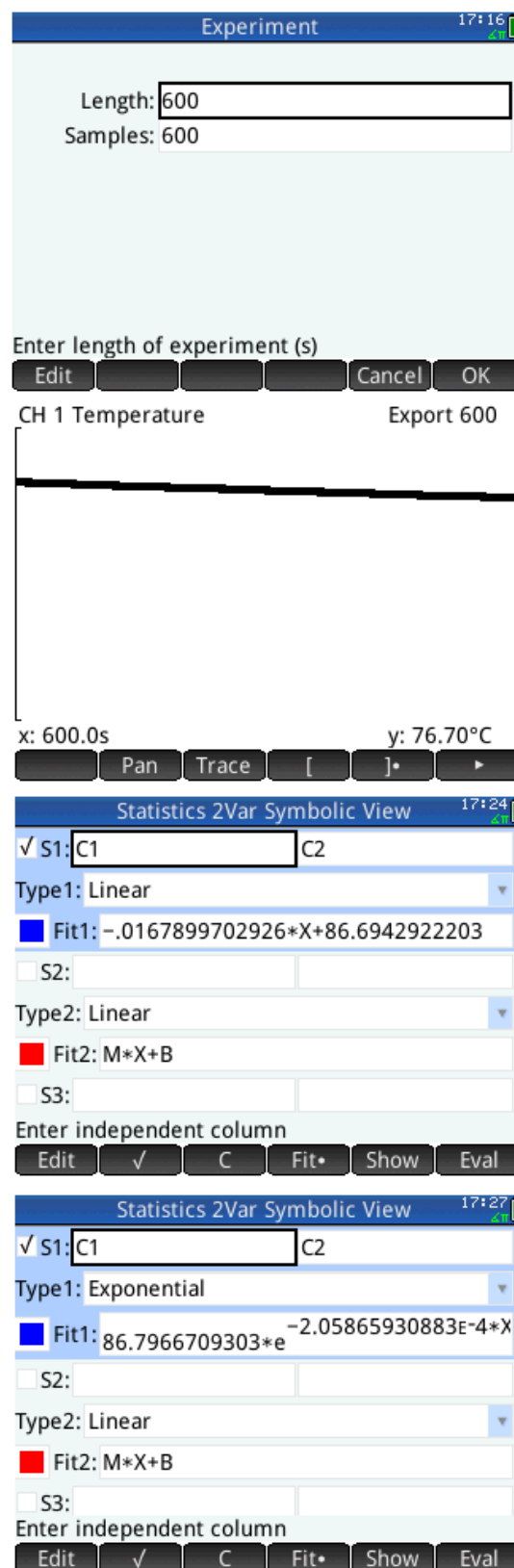
Set up the StreamSmart Unit with a temperature probe (a thermometer). Start the DataStreamer App as before. For this experiment we will set up the length and the number of samples to be collected in advance, this way, the screen will show the whole experiment as it unfolds.

It is important that the thermometer has reached the temperature of the materials being cooled before the experiment starts. You can check by pressing Start in the DataStreamer App and monitoring the temperature to see it is not still rising. When it is not rising, work quickly to start the experiment.

Press the View key. Enter 600 for the experiment length press OK, then 600 for the number of samples and press OK. Now, when everything is ready and set up to start the experiment, press OK again and the data logging should start and you should see a graph begin to appear on screen.

The experiment can be used to keep the students' focus on how we make a mathematical model to describe the rate of cooling, so it is appropriate to take data samples at intervals throughout the experiment. The details of this experiment have been published by the author as Modelling, Functions and Estimation: A Pizza Problem ¹.

When the experiment is finished, the data can be exported to the two variable statistics applet. Press the Export SoftKey then the right Arrow SoftKey, then OK. This will automatically launch the Statistics 2Var App. You can press the Plot key to see your graph again. This cooling 'curve' looks remarkably linear. (The Experiment was done with freshly boiled water in an open cup). If we press the Symb key, we can see a best fit linear model. Then we can select a different model to see if we can improve the quality of the fit. Here, we have chosen an exponential model.



1: Olley, C. (2011) Modelling, Functions and Estimation: A Pizza Problem In, Oldknow, Adrian and Knights, Carol (eds.) *Mathematics Education with Digital Technology*, Continuum, London.

Notes

For further information and educational support contact:

Education Interactive

10 Staplehurst Road

London

SE13 5NB

T: (+44) (0)20 8318 6380

F: (+44) (0)20 8318 6610

info@themathszone.co.uk

www.themathszone.co.uk

